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Mixing-induced CP Violation in the Decay $B_d \to K^0 \bar{K}^0$ within the Standard Model *

Robert Fleischer

Technische Universität München
Physik Department
Institut für Theoretische Physik
D-85748 Garching
Germany

Abstract

Recently, flavour $SU(3)$ symmetry of strong interactions has been combined with certain dynamical assumptions to derive triangle relations among B-meson decay-amplitudes. We show that these relations allow a prediction of the mixing-induced CP asymmetry $\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \to K^0 \bar{K}^0)$. Contrary to statements made in several previous papers, this asymmetry should be non-vanishing in the Standard Model due to QCD-penguins with internal up- and charm-quark exchanges and could be as large as $\mathcal{O}(30\%)$. The branching ratio $\text{BR}(B_d \to K^0 \bar{K}^0)$ is expected to be of $\mathcal{O}(10^{-6})$. In the future, the results presented in this letter should allow interesting tests of the $SU(3)$ triangle relations and of the Standard Model description of CP violation.

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CP-violating asymmetries in neutral B-meson decays are of special interest for an experimental test of the Cabibbo-Kobayashi-Maskawa–model (CKM–model) of CP violation [1]. In contrast to the situation arising in the charged B-meson system, where only *direct* CP violation is present, it is a characteristic feature of the neutral B-meson system that also *mixing-induced* CP violation, which is generated by the interference between $B^0-\bar{B}^0$ mixing and decay processes, may contribute significantly to the CP-violating asymmetries [2]-[8].

The major point of this letter is a prediction of the mixing-induced CP asymmetry of the decay $B_d\to K^0\bar{K}^0$ originating from the generic QCD-penguin process $b\to d\bar{s}s$. The main inputs are the $SU(3)$ flavour symmetry of strong interactions and certain dynamical assumptions to be specified below. In the previous literature, it has been claimed by several authors (see, e.g., refs. [3, 7, 8]) that decays such as $B_d\to K_{\rm S}K_{\rm S}$ or $B_d\to K^0\bar{K}^0$ (the CP asymmetries of both channels are equal) were useful modes to test the Standard Model, since it would predict *zero* CP-violating asymmetries due to a cancellation of weak decay- and mixing-phases. We point out that this statement is only correct, if the QCD-penguin amplitudes are dominated by internal top-quark exchanges. As we shall see, however, QCD-penguins with internal up- and charm-quarks may also play an important role and could lead to rather large CP asymmetries of ${\cal O}(10-50)\%$. Therefore, non-vanishing CP-violating asymmetries measured in $B_d\to K_{\rm S}K_{\rm S}$ (or $B_d\to K^0\bar{K}^0$) would not necessarily give hints to contributions from physics beyond the Standard Model as emphasized, e.g., in ref. [8]. The importance of pure penguin-induced $b\to d\bar{s}s$ modes in respect of direct CP violation has been emphasized previously by Gérard and Hou [9].

If we consider a neutral B_q-decay ($q\in\{d,s\}$) into a CP self-conjugate final state $|f\rangle$, e.g., the transition $B_d\to K^0\bar{K}^0$, the time-dependent and time-integrated CP asymmetries are given by

$$\begin{aligned}a_{\rm CP}(t)\equiv\frac{\Gamma(B^0_q(t)\to f)-\Gamma(\bar{B}^0_q(t)\to f)}{\Gamma(B^0_q(t)\to f)+\Gamma(\bar{B}^0_q(t)\to f)}=\\ {\cal A}^{\rm dir}_{\rm CP}(B_q\to f)\cos(\Delta M_q t)+{\cal A}^{\text{mix-ind}}_{\rm CP}(B_q\to f)\sin(\Delta M_q t)\end{aligned}\tag{1}$$

and

$$\begin{aligned}a_{\rm CP}\equiv\frac{\int_0^\infty dt\left[\Gamma(B^0_q(t)\to f)-\Gamma(\bar{B}^0_q(t)\to f)\right]}{\int_0^\infty dt\left[\Gamma(B^0_q(t)\to f)+\Gamma(\bar{B}^0_q(t)\to f)\right]}=\\ \frac{1}{1+x_q^2}\left[{\cal A}^{\rm dir}_{\rm CP}(B_q\to f)+x_q{\cal A}^{\text{mix-ind}}_{\rm CP}(B_q\to f)\right],\end{aligned}\tag{2}$$

respectively. Here, $\Delta M_q > 0$ is the mass splitting of the physical $B_q^0 - \bar{B}_q^0$ mixing-eigenstates and $x_q \equiv \tau_{B_q}\Delta M_q$ denotes the so-called mixing-parameter. In eqs. (1) and (2), we have separated the *direct* CP-violating contributions characterized by

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_q \to f) \equiv \frac{1-|\xi_f^{(q)}|^2}{1+|\xi_f^{(q)}|^2} \tag{3}$$

from those describing *mixing-induced* CP violation which are proportional to

$$\mathcal{A}_{\rm CP}^{\text{mix-ind}}(B_q \to f) \equiv \frac{2{\rm Im}\xi_f^{(q)}}{1+|\xi_f^{(q)}|^2}. \tag{4}$$

The phase convention independent quantity $\xi_f^{(q)}$ contains essentially all the information needed to evaluate the CP-violating asymmetries. It is given by

$$\xi_f^{(q)} = \exp\left[-i\Theta_{M_{12}}^{(q)}\right]\frac{A(\bar{B}_q^0 \to f)}{A(B_q^0 \to f)}, \tag{5}$$

where $A(\bar{B}_q^0 \to f)$ and $A(B_q^0 \to f)$ are decay amplitudes and

$$\Theta_{M_{12}}^{(q)} = \pi + 2\arg(V_{tq}^* V_{tb}) - \phi_{\rm CP}(B_q) \tag{6}$$

is the $B_q^0 - \bar{B}_q^0$ mixing phase which is a function of the complex phases of the CKM matrix [1]. The phase $\phi_{\rm CP}(B_q)$ arises from our freedom of choosing a CP phase-convention and is defined by the relation $(\mathcal{CP})|B_q^0\rangle = \exp[i\phi_{\rm CP}(B_q)]|\bar{B}_q^0\rangle$. In the convention independent expression (5), $\phi_{\rm CP}(B_q)$ is cancelled by the ratio of decay amplitudes.

If a neutral B_q-meson decay into a final CP eigenstate is dominated by a single CKM-amplitude, $\mathcal{A}_{\rm CP}^{\rm dir}$ vanishes and the uncertainties related to unknown hadronic matrix elements cancel in the mixing-induced asymmetry $\mathcal{A}_{\rm CP}^{\text{mix-ind}}$. In this very interesting case, $\mathcal{A}_{\rm CP}^{\text{mix-ind}}$ is a theoretical clean measure of the angles appearing in the so-called unitarity triangle (for a recent phenomenological analysis of this triangle see, e.g., ref. [10]). An example of such a decay is the channel $B_d \to \psi K_{\rm S}$ which should allow a very clean determination of the angle β in the unitarity triangle from the CP asymmetry $\mathcal{A}_{\rm CP}^{\text{mix-ind}}(B_d \to \psi K_{\rm S}) = -\sin 2\beta$.

On the other hand, if several amplitudes with both different CP-violating weak and CP-conserving strong phases contribute to a neutral B_q-decay, the hadronic uncertainties do not cancel and a theoretical clean prediction of the CP asymmetries (1) and (2) is *a priori* not possible. Furthermore, we expect significant direct CP violation. A decay in this category is, e.g., the pure penguin-induced

mode $B_d \to K^0\bar{K}^0$ [11]. Here, the amplitudes with different weak and strong phases mentioned above arise from QCD-penguins with internal up-, charm- and top-quark exchanges.

After this short introduction, let us now come to our main point. Using $SU(3)$ flavour symmetry of strong interactions [12]-[16] and certain plausible dynamical assumptions (e.g., neglect of annihilation topologies), several triangle relations among B-meson decay amplitudes into $\pi\pi$, πK and $K\bar{K}$ final states have been derived in a recent series of interesting publications [17]-[21]. In this letter, our main intention is to point out that these relations in combination with the recent results of ref. [22] allow an interesting prediction of the mixing-induced CP-violating asymmetry $\mathcal{A}_{\rm CP}^{\text{mix-ind}}(B_d \to K^0\bar{K}^0)$.

To this end, let us use the same notation as in refs. [17]-[22] and denote the amplitudes corresponding to $b \to d$ ($\bar{b} \to \bar{d}$) and $b \to s$ ($\bar{b} \to \bar{s}$) QCD-penguins generically by $\bar{P}$ (P) and $\bar{P}'$ (P'), respectively. Then, taking into account that $(\mathcal{CP})|K^0\bar{K}^0\rangle = +|K^0\bar{K}^0\rangle$ and applying the Wolfenstein parametrization [23] of the CKM-matrix, we obtain

$$\xi^{(d)}_{K^0\bar{K}^0} = -\exp(-i2\beta)\frac{\bar{P}}{P}. \tag{7}$$

In refs. [17]-[21], it has been assumed that the QCD-penguin amplitudes are dominated by internal top-quark exchanges. However, as has been pointed out in [22], sizable contributions may also arise from QCD-penguins with internal up- and charm-quarks. Including these additional penguin amplitudes, which will turn out to be essential for the CP-violating effects arising in the mode $B_d \to K^0\bar{K}^0$, we find [22]

$$\frac{\bar{P}}{P} = \frac{\bar{\rho}_P}{\rho_P}\exp\left[i(2\beta+\psi-\bar{\psi})\right], \tag{8}$$

where

$$\rho_P = \frac{1}{R_t}\sqrt{R_t^2 - 2R_t|\Delta P|\cos(\beta+\delta_{\Delta P}) + |\Delta P|^2} \tag{9}$$

and

$$\tan\psi = \frac{|\Delta P|\sin(\beta+\delta_{\Delta P})}{R_t - |\Delta P|\cos(\beta+\delta_{\Delta P})}. \tag{10}$$

The CP-conjugate quantities $\bar{\rho}_P$ and $\bar{\psi}$ can be obtained easily from (9) and (10) by substituting $\beta \to -\beta$. In eqs. (9) and (10), ΔP describes the contributions of the QCD-penguins with internal u- and c-quarks and is defined by the ratio of strong penguin-amplitudes [22]

$$\Delta P \equiv |\Delta P|\exp(i\delta_{\Delta P}) \equiv \frac{P_c - P_u}{P_t - P_u}, \tag{11}$$

whereas the CKM-factor

$$R_t \equiv \frac{1}{\lambda}\frac{|V_{td}|}{|V_{cb}|} \tag{12}$$

represents one side of the unitarity triangle. Present experimental data imply that R_t is of $\mathcal{O}(1)$ [10]. Note that ΔP is affected strongly by hadronic uncertainties, in particular by unknown strong final state interaction phases. In the limit of degenerate u- and c-quark masses, ΔP would vanish due to the GIM mechanism. However, since $m_u \approx 4.5$ MeV, whereas $m_c \approx 1.3$ GeV, this GIM cancellation is incomplete and in principle sizable effects arising from ΔP could be expected [22].

Combining (7) and (8) gives the expression

$$\xi^{(d)}_{K^0\bar{K}^0} = -\frac{\bar{\rho}_P}{\rho_P}\exp\left[i(\psi-\bar{\psi})\right]. \tag{13}$$

If we consider only QCD-penguins with internal top-quark exchanges corresponding to $\Delta P = 0$ [17]-[21], the weak decay- and mixing-phases cancel each other and we get

$$\xi^{(d)}_{K^0\bar{K}^0}(\Delta P = 0) = -1. \tag{14}$$

Consequently, the CP asymmetries (1) and (2) vanish in this approximation. In several previous papers (see, e.g., ref. [8]), it has been claimed that this result would provide a test of the Standard Model and that observed non-vanishing CP asymmetries would indicate physics beyond the Standard Model. However, within the Standard Model, non-vanishing asymmetries $\mathcal{A}^{\text{dir}}_{\text{CP}}(B_d \to K^0\bar{K}^0)$ and $\mathcal{A}^{\text{mix-ind}}_{\text{CP}}(B_d \to K^0\bar{K}^0)$ may arise from QCD-penguin contributions with internal u- and c-quarks.

In order to illustrate this statement quantitatively, let us apply, as in ref. [22], the perturbative approach of Bander, Silverman and Soni [24] – the so-called "BSS-mechanism" – to estimate ΔP. We will also investigate the expected magnitude of the "average" branching ratio $\text{BR}(B_d \to K^0\bar{K}^0)$ defined by

$$\begin{aligned}\text{BR}(B_d \to K^0\bar{K}^0) \;\equiv\; & \frac{1}{2}\int_0^\infty dt\left[\Gamma(B^0_d(t)\to K^0\bar{K}^0)+\Gamma(\bar{B}^0_d(t)\to K^0\bar{K}^0)\right] = \\ & \frac{1}{2}\left(1+|\xi^{(d)}_{K^0\bar{K}^0}|^2\right)\Gamma(B^0_d\to K^0\bar{K}^0)\tau_{B_d}.\end{aligned} \tag{15}$$

Here, $\Gamma(B^0_d \to K^0\bar{K}^0)$ denotes the transition rate of the decay $B^0_d \to K^0\bar{K}^0$ which can be obtained by performing the usual phase-space integrations.

To simplify the discussion, we neglect the renormalization group evolution from $\mu = \mathcal{O}(M_W)$ down to $\mu = \mathcal{O}(m_b)$ and take into account QCD renormalization effects only approximately through the substitution $\alpha_s \to \alpha_s(\mu)$ (for a discussion of QCD-corrections affecting $B_d \to K^0\bar{K}^0$, see ref. [11]). Then, choosing

$R_t = 1$ and various angles β, we find the curves shown in Figs. 1–3 describing the dependences of $\mathcal{A}_{\rm CP}^{\rm mix\text{-}ind}(B_d \to K^0\bar{K}^0)$, $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to K^0\bar{K}^0)$ and of the corresponding time-integrated asymmetry $a_{\rm CP}$, respectively, on the momentum transfer k^2 of the gluons appearing in the usual QCD-penguin diagrams. Applying, in addition to the BSS-mechanism [24], the *factorization* approximation and the form factors presented by Bauer, Stech and Wirbel [25] to estimate the relevant hadronic matrix elements, we obtain the curves for $\mathrm{BR}(B_d \to K^0\bar{K}^0)$ depicted in Fig. 4. The details of the calculation of ΔP within the perturbative BSS-mechanism can be found in ref. [22], whereas the evaluation of the branching ratio $\mathrm{BR}(B_d \to K^0\bar{K}^0)$ has been outlined in ref. [11]. In drawing Figs. 1–4, we have taken into account that the present range of $|V_{ub}/V_{cb}|$ implies $\beta \lesssim 45^\circ$ [10]. Looking at these figures and choosing k^2 to lie within the "physical" range

$$\frac{1}{4} \lesssim \frac{k^2}{m_b^2} \lesssim \frac{1}{2}, \tag{16}$$

which follows from simple kinematical considerations at the quark level, we expect rather large asymmetries of the order $(10-50)\%$, which are quite promising from the experimental point of view, and $\mathrm{BR}(B_d \to K^0\bar{K}^0) = \mathcal{O}(10^{-6})$. We are aware of the fact that the numerical estimates given here are very rough. They illustrate, however, the expected orders of magnitude.

After this short quantitative illustration, let us now turn to the prediction of the mixing-induced CP asymmetry $\mathcal{A}_{\rm CP}^{\rm mix\text{-}ind}(B_d \to K^0\bar{K}^0)$. Using two rates that determine $|P|$ and $|P'|$, e.g., those of the modes $B^+ \to K^+\bar{K}^0$ and $B^+ \to \pi^+K^0$, respectively, and the triangle relations [19]-[21]

$$\begin{array}{rcl} A(B_d^0 \to \pi^+\pi^-) + \sqrt{2}A(B_d^0 \to \pi^0\pi^0) & = & \sqrt{2}A(B^+ \to \pi^+\pi^0) \\ (T+P) \qquad + \qquad (C-P) & = & (T+C) \end{array} \tag{17}$$

and

$$\begin{array}{rcl} A(B_d^0 \to \pi^-K^+)/r_u + \sqrt{2}A(B_d^0 \to \pi^0K^0)/r_u & = & \sqrt{2}A(B^+ \to \pi^+\pi^0) \\ (T+P'/r_u) \qquad + \qquad (C-P'/r_u) & = & (T+C), \end{array} \tag{18}$$

where $r_u = V_{us}/V_{ud}$ and T (C) refers to the "tree" ("colour-suppressed") amplitude of the decay $B^+ \to \pi^+\pi^0$, the relative angle ϑ between P and P' can be measured. In contrast to the assertions made in [19]-[21], ϑ is not equal to the CKM-angle β, but receives some hadronic corrections [22]:

$$\vartheta = \beta + \psi - \psi'. \tag{19}$$

Here, ψ' is a pure strong phase that is given by [22]

$$\tan\psi' = \frac{|\Delta P|\sin\delta_{\Delta P}}{1-|\Delta P|\cos\delta_{\Delta P}}. \tag{20}$$

If we consider in addition the corresponding CP-conjugate processes, the angle

$$\bar{\vartheta} = -\beta + \bar{\psi} - \psi' \tag{21}$$

can be determined as well. Note that $\bar{\psi}' = \psi'$, since no weak phases are present in (20). Consequently, combining (19) and (21) appropriately, we find

$$2\beta + \bar{\vartheta} - \vartheta = \bar{\psi} - \psi \tag{22}$$

and $\xi^{(d)}_{K^0\bar{K}^0}$ can be expressed as

$$\xi^{(d)}_{K^0\bar{K}^0} = -\frac{|\bar{P}|}{|P|}\exp\left[-i(2\beta+\bar{\vartheta}-\vartheta)\right] \tag{23}$$

which yields

$$\mathcal{A}^{\rm dir}_{\rm CP}(B_d\to K^0\bar{K}^0) = \frac{|P|^2-|\bar{P}|^2}{|P|^2+|\bar{P}|^2} \tag{24}$$

and

$$\mathcal{A}^{\text{mix-ind}}_{\rm CP}(B_d\to K^0\bar{K}^0) = \frac{2|P||\bar{P}|\sin(2\beta+\bar{\vartheta}-\vartheta)}{|P|^2+|\bar{P}|^2}. \tag{25}$$

Note that eq. (24) implies that $\mathcal{A}^{\rm dir}_{\rm CP}(B_d\to K^0\bar{K}^0)$ should be equal to the CP-violating asymmetry

$$a_{\rm CP}(B^\pm\to K^\pm K^0) \equiv \frac{\Gamma(B^+\to K^+\bar{K}^0)-\Gamma(B^-\to K^-K^0)}{\Gamma(B^+\to K^+\bar{K}^0)+\Gamma(B^-\to K^-K^0)}. \tag{26}$$

The expression (25) for the mixing-induced CP asymmetry $\mathcal{A}^{\text{mix-ind}}_{\rm CP}(B_d\to K^0\bar{K}^0)$ is interesting in two respects:

i) If 2β is known, the triangle relations (17) and (18) (and those of the corresponding CP-conjugate processes) allow a prediction of the CP-violating asymmetry $\mathcal{A}^{\text{mix-ind}}_{\rm CP}(B_d\to K^0\bar{K}^0)$. In principle, the angle β can be determined from the $SU(3)$ triangle relations as well [19]-[21]. However, irrespectively of $SU(3)$-breaking effects and certain neglected diagrams (annihilation topologies, etc.), the QCD-penguin contributions with internal u- and c-quarks preclude a clean determination of β by using the branching ratios only (see eqs. (19) and (21)). As has been pointed out in ref. [22], this difficulty can be overcome by measuring in addition the ratio x_d/x_s of $B^0_d-\bar{B}^0_d$

to $B_s^0-\bar{B}_s^0$ mixings to obtain the CKM-parameter R_t. The probably cleanest way of determining 2β is the measurement of the mixing-induced CP asymmetry $\mathcal{A}_{\rm CP}^{\mbox{\scriptsize mix-ind}}(B_d\to\psi K_{\rm S})$. An interesting possibility to extract $\sin 2\beta$ from the two branching ratios $\mbox{BR}(K^+\to\pi^+\nu\bar\nu)$ and $\mbox{BR}(K_{\rm L}\to\pi^0\nu\bar\nu)$ has been proposed recently by Buchalla and Buras [26].

ii) If one determines $\mathcal{A}_{\rm CP}^{\mbox{\scriptsize mix-ind}}(B_d\to K^0\bar K^0)$ from measurements of the corresponding CP-violating asymmetries and $\bar\vartheta$, ϑ from the two-triangle constructions outlined in refs. [19]-[21], the hadronic uncertainties arising in (19) and (21) from QCD-penguins with internal u- and c-quarks affecting the extraction of the CKM-angle β can be eliminated and another clean determination of β is possible (up to corrections related to $SU(3)$-breaking and certain neglected diagrams).

In summary, we have shown that the CP-violating asymmetries $\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\bar K^0)$ and $\mathcal{A}_{\rm CP}^{\mbox{\scriptsize mix-ind}}(B_d\to K^0\bar K^0)$ are generated within the framework of the Standard Model by QCD-penguins with internal up- and charm-quark exchanges and are, thus, interesting quantities to obtain experimental insights into the physics of these contributions. Estimates obtained by applying the perturbative approach of Bander, Silverman and Soni give rather promising asymmetries of the order $(10-50)\%$ depending strongly on the angle β and branching ratios at the 10^{-6} level. Therefore, measured non-vanishing CP asymmetries would not necessarily imply physics beyond the Standard Model as claimed in several previous papers.

While the direct CP-violating asymmetry $\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\bar K^0)$ should be equal to the one arising in the charged B-decay $B^+\to K^+\bar K^0$, the mixing-induced CP asymmetry $\mathcal{A}_{\rm CP}^{\mbox{\scriptsize mix-ind}}(B_d\to K^0\bar K^0)$ can be predicted by using triangle relations among B-meson decay-amplitudes which follow from the $SU(3)$ flavour symmetry of strong interactions and certain dynamical assumptions. These assumptions consist, e.g., of neglecting annihilation-like topologies. Uncertainties related to $SU(3)$-breaking effects have been discussed in [17]-[22]. Additional corrections to the triangle relations could also arise from electroweak penguins which may, in the presence of a heavy top-quark, lead to sizable contributions to the penguin sectors of B-decays into final states containing mesons with CP-self-conjugate quark contents [27]-[29]. Therefore, experimental tests of the validity of the triangle relations (17) and (18) are desirable.

In the future, when it will hopefully be possible to measure CP-violating asymmetries arising in the decay $B_d\to K^0\bar K^0$, the results presented in this letter

should provide such a test of the $SU(3)$ triangle relations and, moreover, of our understanding of CP violation.

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Figure Captions

Fig. 1: The dependence of $\mathcal{A}_{\rm CP}^{\rm mix\text{-}ind}(B_d \to K^0\bar{K}^0)$ on k^2/m_b^2 for $R_t = 1$ and various values of the CKM-angle β.

Fig. 2: The dependence of $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to K^0\bar{K}^0)$ on k^2/m_b^2 for $R_t = 1$ and various values of the CKM-angle β.

Fig. 3: The dependence of the time-integrated CP asymmetry $a_{\rm CP}$ of the decay $B_d \to K^0\bar{K}^0$ on k^2/m_b^2 for $R_t = 1$ and various values of the CKM-angle β. ($x_d = 0.7$)

Fig. 4: The dependence of the "average" branching ratio $\mathrm{BR}(B_d \to K^0\bar{K}^0)$ defined by eq. (15) on k^2/m_b^2 for $R_t = 1$ and various values of the CKM-angle β. ($\tau_{B_d} = 1.6$ ps, $|V_{cb}| = 0.04$, $\Lambda_{\overline{\rm MS}}^{(4)} = 0.3$ GeV)

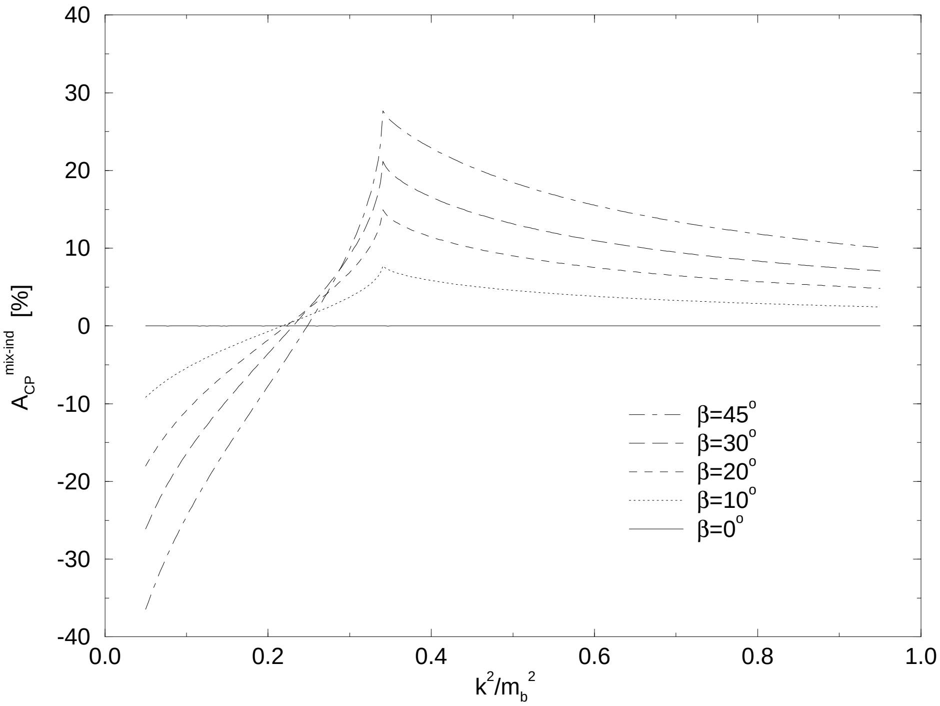

Figure 1:

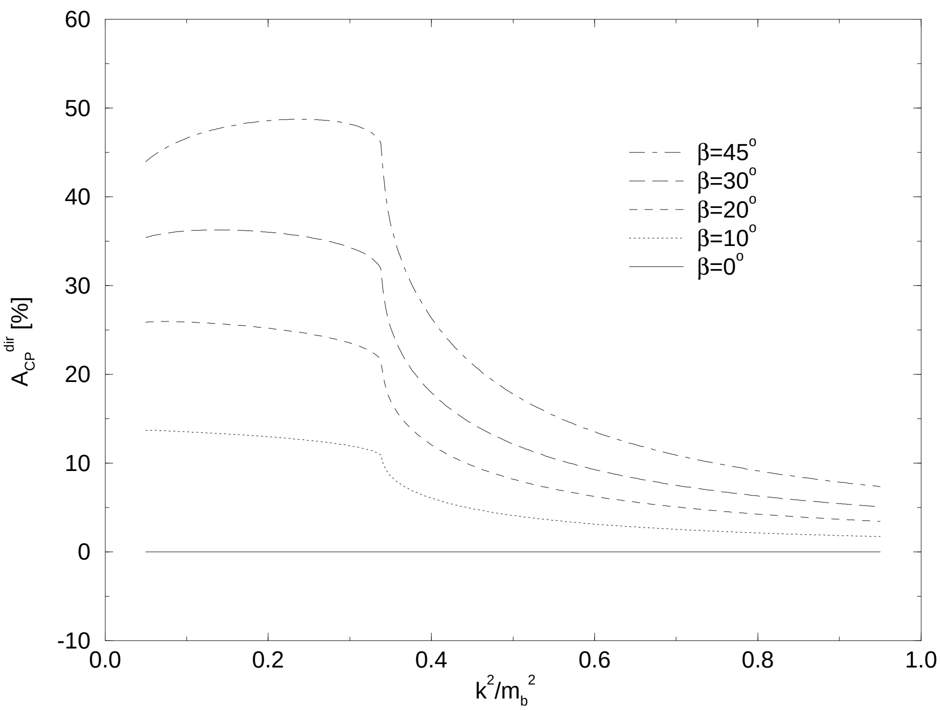

Figure 2:

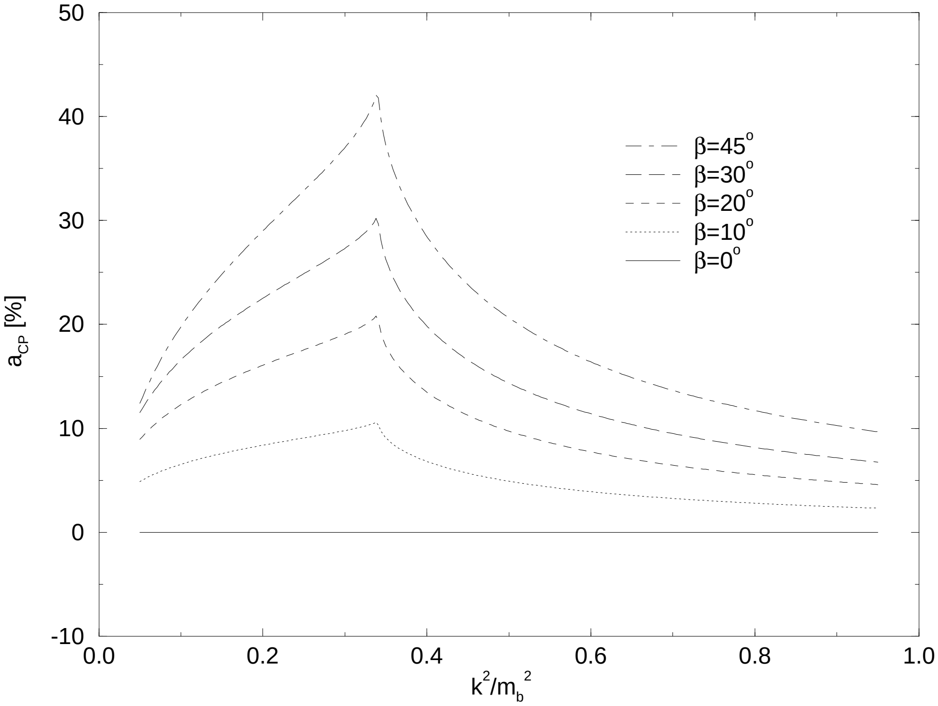

Figure 3:

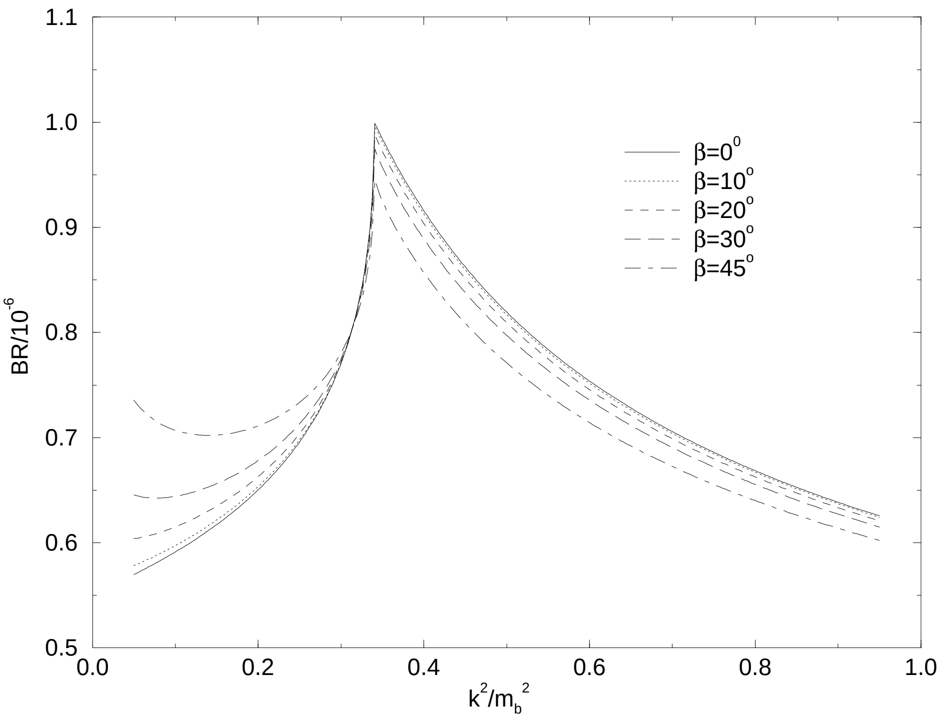

Figure 4: